

8300

GCSE MATHEMATICS

Foundation and Higher tier – Questions by Topic – Mark Scheme

Ratio, Proportion and Rates of Change

Writing ratios in the form 1:n or n:1

v1.0

Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt

M	Method marks are awarded for a correct method which could lead to a correct answer.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied
B	Marks awarded independent of method.
ft	Follow through marks. Marks awarded for correct working following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
M dep	A method mark dependent on a previous method mark being awarded.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
oe	Or equivalent. Accept answers that are equivalent. e.g. accept 0.5 as well as $\frac{1}{2}$
[a, b]	Accept values between a and b inclusive.
3.14...	Allow answers which begin 3.14 e.g. 3.14, 3.142, 3.1416
Use of brackets	It is not necessary to see the bracketed work to award the marks.

Examiners should consistently apply the following principles

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Responses which appear to come from incorrect methods

Whenever there is doubt as to whether a student has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the student. In cases where there is no doubt that the answer has come from incorrect working then the student should be penalised.

Questions which ask students to show working

Instructions on marking will be given but usually marks are not awarded to students who show no working.

Questions which do not ask students to show working

As a general principle, a correct response is awarded full marks.

Misread or miscopy

Students often copy values from a question incorrectly. If the examiner thinks that the student has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

Further work

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

Choice

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

Work not replaced

Erased or crossed out work that is still legible should be marked.

Work replaced

Erased or crossed out work that has been replaced is not awarded marks.

Premature approximation

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

Q1.

3

B1

[1]

Q2.

$\frac{3}{4}$

B1

[1]

Q3.

0.5

or

$\frac{2 \times 10^3}{5 \times 10^{-1}}$ or $\frac{\text{their 2000}}{5 \times 10^{-1}}$

their 2000 from part (a)

or

$0.4 \times 10^{3 - (-1)}$ or 0.4×10^4

oe

M1

4000 or 4×10^3

ft 2 × their 2000 in part (a)

A1ft

[2]

Q4.

(a) 28 : 12 or 14 : 6

or

$56 \div 8$ and $24 \div 8$

(may be done in stages)

or

3 and 7 seen

M1

7 : 3

A1

(b) 1.25 : 1

oe e.g. $\frac{5}{4} : 1$

B1

(c) $180 \div (1 + 9)$ or 18 or 162

M1

18 and 162

A1

Additional Guidance

162 and 18

M1A0

Build-up method will score 2 or 0

e.g. 1 : 9

2 : 18 does not score M1 for 18

[5]

Q5.

$$3^{11} (: 3^7)$$

or

$$3^6 : 3^2$$

or

$$3^5 : 3^{(1)}$$

or

$$\frac{a}{3^7}$$

or

$$177\,147 : 2187$$

oe eg $729 : 9$ or $243 : 3$

3^n may be implied by a multiplication string of n 3s

a can be any value other than 3^7

M1

$$\frac{3^{11}}{3^7} (: 1)$$

or

$$\frac{3^6}{3^2}$$

$$3^2 (: 1) \text{ or } 3^6 \times 3^{-2} (: 1)$$

or

$$\frac{3^5}{3^{(1)}}$$

$$3^{(1)} (: 1) \text{ or } 3^{-1} \times 3^5 (: 1)$$

or

$$3^4 (: 1)$$

or

$$\frac{177\,147}{2187} (: 1)$$

oe left-hand side with one or two components

$$\text{eg } \frac{729}{9} : 1$$

or

$$243 \times \frac{1}{3} : 1$$

allow $(: 1)$ to be $(: 3^0)$

3^n may be implied by a multiplication string of n 3s

M1dep

$$81 : 1$$

A1

Additional Guidance

$$\frac{3^6 \times 3^5}{3^7}$$

$(: 1)$ with no further work

M1M0A0

$81 : 1$ or $3^4 (: 1)$ could be seen from incorrect working

$$\text{eg } \frac{9^{11}}{3^7} = 3^4$$

Answer $81 : 1$

M1M0A0

[3]

Q6.

$$\frac{100 - 68}{2} \text{ or } \frac{32}{2} \text{ or } 16(\%)$$

or

$$\frac{1 - 0.68}{2} \text{ or } \frac{0.32}{2} \text{ or } 0.16$$

oe

M1

$$68 : 16$$

oe ratio not in form n : 1

eg 68% : 16% or 17 : 4 or 0.68 : 0.16

or

$$\frac{68}{16} \text{ or } 68 \div 16 \text{ or } 4.25$$

*oe fraction or division or decimal
implied by 4.25 : 1 oe*

A1

$$4.25 : 1 \text{ or } 4\frac{1}{4} : 1$$

*oe ratio in form n : 1 eg $\frac{68}{16} : 1$
ft any **ratio** not in form n : 1
ft values must give n to 2 dp or better*

Additional Guidance

$$\frac{100 - 68}{2} = 66$$

M1

$$68 : 66 = 1.03 : 1$$

A0B1ft

$$68 : 32 = 2.125 : 1 \text{ or } 68 : 32 = 2.13 : 1$$

M0A0B1ft

$$68 \div 32, \text{ Answer } 2.125 : 1 \text{ (no ratio seen to ft)}$$

M0A0B0ft

Correct ratio with subsequent truncation or rounding to < 2 dp

$$\text{eg1 } 4.25 : 1, \text{ Answer } 4 : 1$$

M1A1B0

$$\text{eg2 } 68 : 32 = 2.125 : 1, \text{ Answer } 2.1 : 1$$

M0A0B0

$$4.25n : 1$$

M1A1B0

$$16 : 1 \text{ with no other creditworthy work}$$

M1A0B0

[3]

Q7.

2.25 or $2\frac{1}{4}$ or $\frac{9}{4}$
oe

B1

Additional Guidance

Condone eg 1 : 2.25

B1

[1]

Q8.

$\frac{30}{12}$ or 2.5

oe fraction, mixed number or decimal

eg $\frac{5}{2}$ or $2\frac{1}{2}$

B1

Additional Guidance

Ignore simplification or conversion attempt after correct answer seen

eg $\frac{30}{12}$ in working with 2.6 on answer line

B1

30 ÷ 12 with no further correct work

B0

[1]

Q9.

(a) **Alternative method 1**

$$\frac{8}{5} \text{ and } \frac{5}{5}$$

or

any correct ratio using integers

or

$$\frac{1.6}{1.6+1} \text{ or } \frac{1.6}{2.6}$$

*oe fractions with common denominators
eg 16 : 10*

$$\frac{8}{13}$$

M1

$$\text{oe fraction eg } \frac{4000}{6500}$$

A1

Alternative method 2

$$6500 \div (1.6 + 1) \text{ or } 2500$$

or

$$6500 \div (1.6 + 1) \times 1.6 \text{ or } 4000$$

$$\text{or } \frac{2500}{6500} \text{ or } \frac{5}{13} \text{ or } \frac{1}{2.6}$$

oe

M1

$$\frac{8}{13}$$

$$\text{oe fraction eg } \frac{4000}{6500}$$

A1

$$(b) \quad 1 : 0.625 \text{ or } 1 : \frac{5}{8}$$

oe fraction

B1

Additional Guidance

0.625 in working

1 : 0.6

B0

[3]

Q10.

$912 \div 15.2$ or 60
oe implied by 62

M1

$912 \div (\text{their } 60 + 2)$
or $912 \div 62$
or $[14.7, 14.71]$
oe

M1dep

$[14.7, 14.71] : 1$
or $15 : 1$ with M2 awarded
oe eg $\frac{456}{31} : 1$

A1

Additional Guidance

M1 may be awarded for correct work, with no answer or incorrect answer, even if this is seen amongst multiple attempts

Do not allow misreads for 15.2

[3]

Q11.

$1 : 1.3$

B1

[1]

Q12.

(a) $120 \div (1 + 4)$ or $120 \div 5$ or 24 or 96
oe

M1

$24 : 96$

in order

A1

Additional Guidance

$96 : 24$

M1A0

$120 \div 5$ and $120 \div 4$ is choice unless intention is clear

M0A0

Further cancelling after $24 : 96$ seen e.g. $1 : 4$

M1A0

(b) $1.75 : 1$ or $1\frac{3}{4} : 1$ or $\frac{7}{4} : 1$

B1

[3]

Q13.

8

B1

$$\frac{1}{0.4} \text{ or } \frac{10}{4} \text{ or } 2.5$$
$$\text{or } \frac{1}{\frac{2}{5}} \text{ or } \frac{5}{2} \text{ or } 2\frac{1}{2}$$

8×0.4 or 3.2 implies B1M1

$16 : 5$ or equivalent ratio implies B1M1

M1

3.2 : 1

$$\text{or } \frac{16}{5} : 1 \text{ or } 3\frac{1}{5} : 1$$

ft B0M1

A1ft

Additional Guidance

$8^3 = 512$ or $8 \times 8 \times 8 = 512$ alone is not sufficient for B1

ft answers must have n exact or correctly rounded to at least 2 sf

eg $\sqrt{512} = 22.62$ (incorrect and truncated)

B0

2.5

M1

9.05 : 1

A1ft

ft answer exact surd value

$$\text{eg } \sqrt{512} = 16\sqrt{2}$$

B0

2.5

M1

$$9.05 : 1 \text{ or } \frac{32}{5}\sqrt{2} : 1$$

A1ft

[3]

Q14.

$$2 \times \pi \times 8 \times 22 \text{ or } 352\pi$$

or [1105, 1106]

Area of lampshade A

oe

$$2 \times \pi \times 0.08 \times 0.22 \text{ or } 0.0352\pi$$

or [0.1105, 0.1106]

M1

$$4 \times \frac{1}{2} \times 15 \times 24 \text{ or } 720$$

Area of lampshade B

oe

$$4 \times \frac{1}{2} \times 0.15 \times 0.24 \text{ or } 0.072$$

M1

$$\text{their } 352\pi \div 100^2 \times 400$$

or

$$14.08\pi$$

or

$$[44.2, 44.24]$$

their 0.0352\pi \times 400

or

$$14.08\pi$$

or

$$[44.2, 44.24]$$

dep on 1st M1

$$\text{their } 720 \div 100^2 \times 400$$

or

$$28.8(0)$$

their 0.072 \times 400

or

$$28.8(0)$$

dep on 2nd M1

M1dep

$$\text{their } 14.08\pi + 3.50$$

$$\text{or } [47.7, 47.74]$$

and

$$\text{their } 28.8(0) + 7.5(0) \text{ or } 36.3(0)$$

dep on M3 and method for fabric cost for both lampshades correct

M1dep

$$1.3(1...) : 1 \text{ or } 1.32 : 1$$

A1

Additional Guidance

$$1 : 1.3(1...) \text{ or } 1 : 1.32$$

M4A0

[5]

Q15.

Alternative method 1

$$4.5 \div 2 \text{ or } 2.25$$

M1

$$\sqrt{\text{their } 2.25} \text{ or } 1.5$$

M1dep

$$1 : 1.5 \text{ or } 1 : 1\frac{1}{2} \text{ or } 1 : \frac{3}{2}$$

A1

Alternative method 2

$$\sqrt{2} : \sqrt{4.5}$$

M1

$$1 : \frac{\sqrt{\text{their } 4.5}}{\sqrt{\text{their } 2}}$$

M1dep

$$1 : 1.5 \text{ or } 1 : 1\frac{1}{2} \text{ or } 1 : \frac{3}{2}$$

A1

Alternative method 3

$$\sqrt{4} : \sqrt{9}$$

M1

$$2 : 3$$

M1dep

$$1 : 1.5 \text{ or } 1 : 1\frac{1}{2} \text{ or } 1 : \frac{3}{2}$$

A1

Alternative method 4

$$2 \div 4.5 \text{ or } \frac{4}{9} \text{ or } 0.\dot{4}$$

accept 0.44 or better

A1

$$\sqrt{\text{their } \frac{4}{9}} \text{ or } \frac{2}{3} \text{ or } 0.\dot{6}$$

and

$$\frac{2}{3} : 1$$

accept 0.66 or better

M1dep

$$1 : 1.5 \text{ or } 1 : 1\frac{1}{2} \text{ or } 1 : \frac{3}{2}$$

A1

Additional Guidance

1 : 1.5 on answer line with no evidence of incorrect method

M1M1A1

[3]

Q16.

Alternative method 1 – using angles around O and angles inside arrowhead

$$ACO = 90 - 83 \text{ or } ACO = 7$$

may be seen on diagram

M1

$$\text{Acute } BOC = 2 \times 28$$

$$\text{or acute } BOC = 56$$

may be seen on diagram

M1

$$\text{Reflex } BOC = 360 - \text{their } 56$$

$$\text{or reflex } BOC = 304$$

may be seen on diagram

dep on 2nd M1

M1dep

$$ABO = 360 - \text{their } 304 - \text{their } 7 - 28$$

or

$$ABO = 21$$

may be seen on diagram

dep on M3

M1dep

$$ABO = 21 \text{ and } ACO = 7$$

$$\text{and } 21 : 7 = 3 : 1$$

all angle values must be seen

A1

Alternative method 2 – with line OA added

$$ACO = 90 - 83 \text{ or } ACO = 7$$

may be seen on diagram

M1

$$OAC = 7$$

or

$$ABO + ACO = 28$$

may be seen on diagram

M1dep

$$OAB = 28 - 7 \text{ Or } OAB = 21$$

or

$$ABO = 28 - 7$$

may be seen on diagram

dep on M2

M1dep

$$ABO = 21$$

may be seen on diagram

dep on M3

M1dep

$$ABO = 21 \text{ and } ACO = 7$$

$$\text{and } 21 : 7 = 3 : 1$$

all angle values must be seen

A1

Alternative method 3 – using alternate segment theorem

$$ACO = 90 - 83 \text{ or } ACO = 7$$

may be seen on diagram

M1

$$\text{Acute } BOC = 2 \times 28$$

$$\text{or acute } BOC = 56$$

may be seen on diagram

M1

$$ABC = 83$$

may be seen on diagram

M1

$$OBC = \frac{180 - \text{their } 56}{2}$$

$$\text{or } OBC = 62$$

and

$$ABO = 83 - \text{their } 62 \text{ or } ABO = 21$$

may be seen on diagram

dep on 2nd and 3rd M1

M1dep

$$ABO = 21 \text{ and } ACO = 7$$

$$\text{and } 21 : 7 = 3 : 1$$

all angle values must be seen

A1

Alternative method 4 – using triangles OBC and ABC

$$ACO = 90 - 83 \text{ or } ACO = 7$$

may be seen on diagram

M1

$$\text{Acute } BOC = 2 \times 28$$

$$\text{or acute } BOC = 56$$

may be seen on diagram

M1

$$\text{Acute } BOC = 2 \times 28$$

$$\text{or acute } BOC = 56$$

may be seen on diagram

M1

$$OBC = \frac{180 - \text{their } 56}{2}$$

$$\text{or } OBC = 62$$

may be seen on diagram

or angle OCB

dep on 2nd M1

$$ABO = 180 - 28 - 62 - 62 - 7$$

$$\text{or } ABO = 21$$

oe

may be seen on diagram

dep on M3

M1dep

$$ABO = 21 \text{ and } ACO = 7$$

$$\text{and } 21 : 7 = 3 : 1$$

all angle values must be seen

A1

Additional Guidance

If angles are not correctly positioned on the diagram they must be correctly identified in the working, eg $\text{BOC} = 56$ is M0 if not correctly positioned on the diagram and not identified as acute

$\text{ACO} = 7$ and $\text{ABO} : \text{ACO} = 21 : 7$ with no other correct working

M1M0M0M0A0**[5]**